



THE AUSTRALIAN NATIONAL UNIVERSITY

# **Brlcc – recent developments in theoretical conversion coefficients**

**T. Kibédi (ANU)**

Brlcc status

Conversion coefficients for superheavy elements

Electronic factors for E0 transitions

Collaborators

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A.E. Stuchbery and D. Tsifakis (ANU)

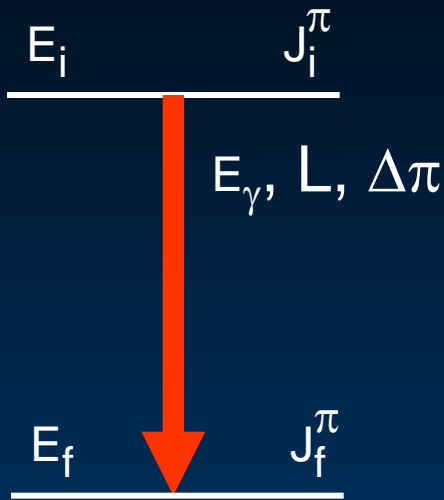
## Energetics of $\gamma$ -decay:

$$E_i = E_f + E_\gamma + T_r$$

$$0 = p_r + p_\gamma,$$

where  $T_r = (p_r)^2/2M$ ; usually  $T_r/E_\gamma \sim 10^{-5}$

## Angular momentum and parity selection rules; multipolarities



Multipolarity known

$\Delta J$  may not be unique

unique  $\Delta\pi$

$$|J_i - J_f| \leq L \leq J_i + J_f; L \neq 0$$

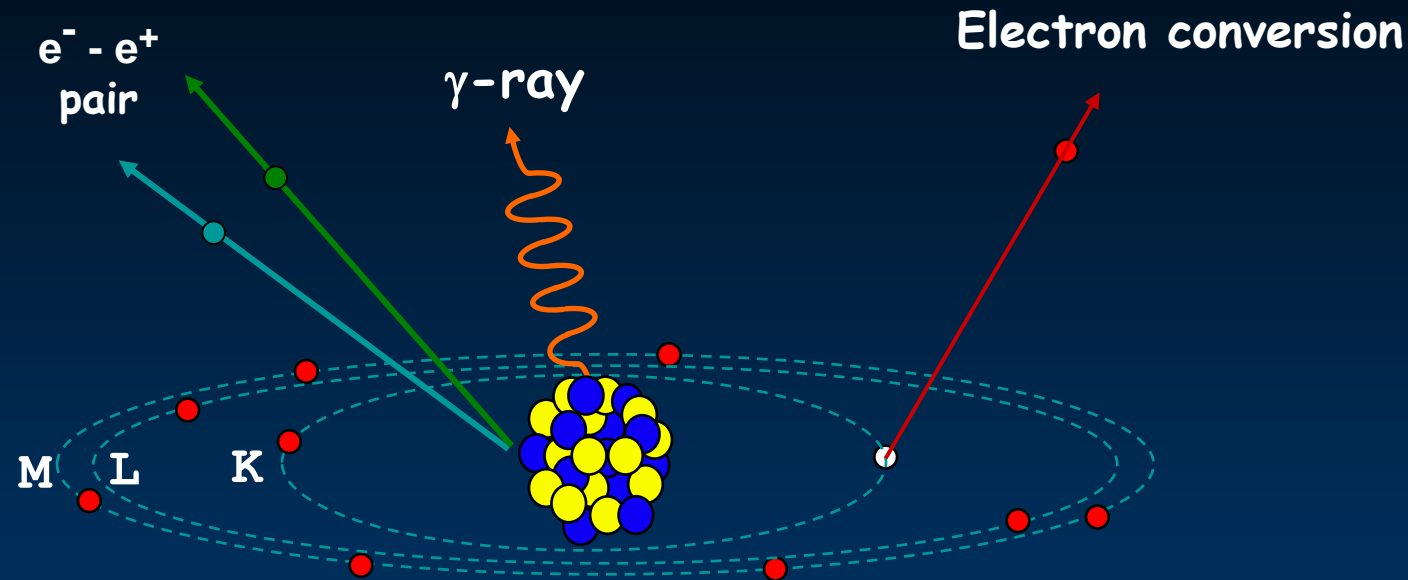
$$J_i = J_f$$

$$\Delta\pi = \text{no}; \quad E2, E4, E6 \quad M1, M3, M5 \quad E0$$

$$\Delta\pi = \text{yes}; \quad E1, E3, E5 \quad M2, M4, M6$$

## Mixed multipolarity

$$\delta^2(\pi`L`) = I_\gamma(\pi`L`) / I_\gamma(\pi L)$$



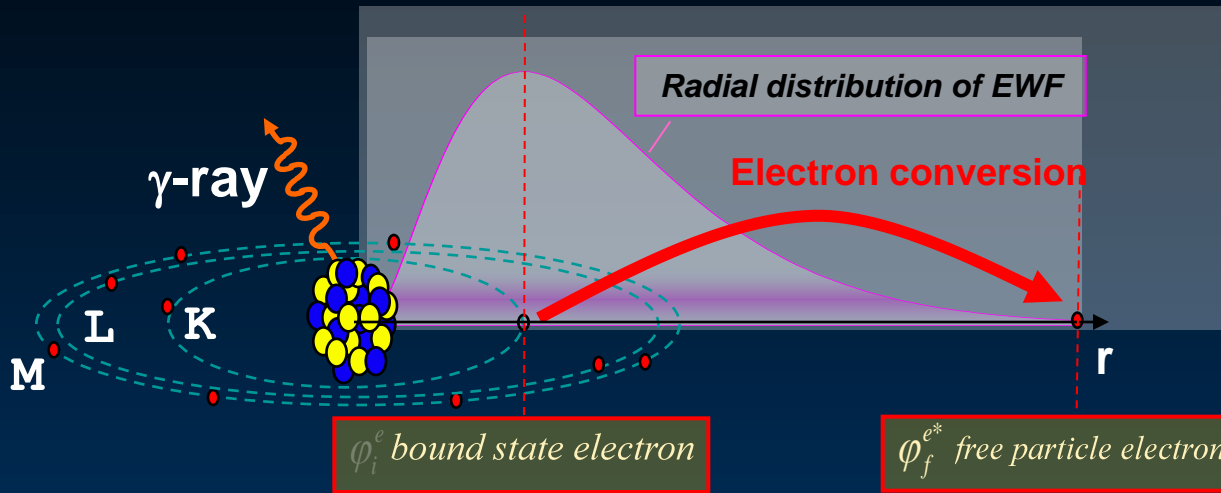
*Transition probability*

$$\lambda_T = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \dots + \lambda_\pi$$

*Conversion Coefficient*

$$\alpha_{ce,\pi} = \lambda_{ce,\pi} / \lambda_\gamma$$

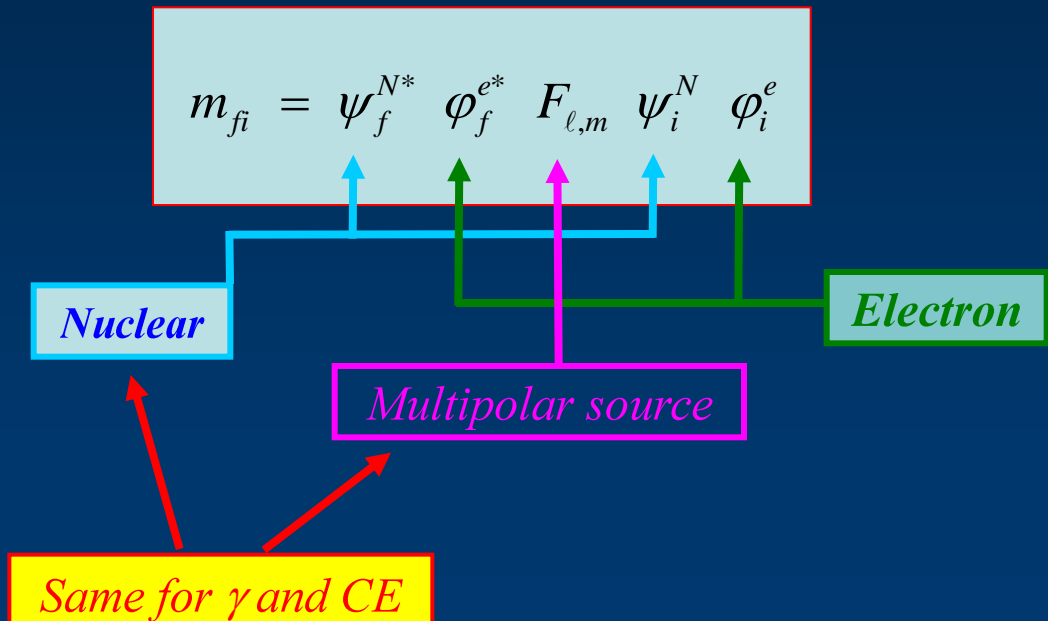
# Conversion electron process and electromagnetic interaction



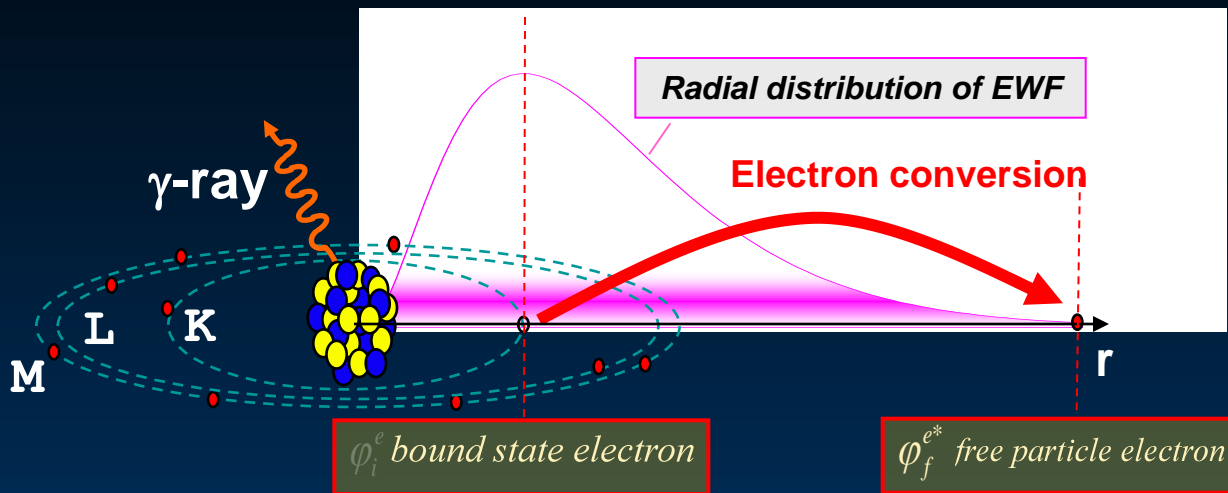
**Fermi's golden rule**

$$\alpha_e = \frac{\lambda_e}{\lambda_\gamma} \Rightarrow \lambda_e = \frac{2\pi}{\hbar} |m_{fi}|^2 \frac{d\rho}{dE}$$

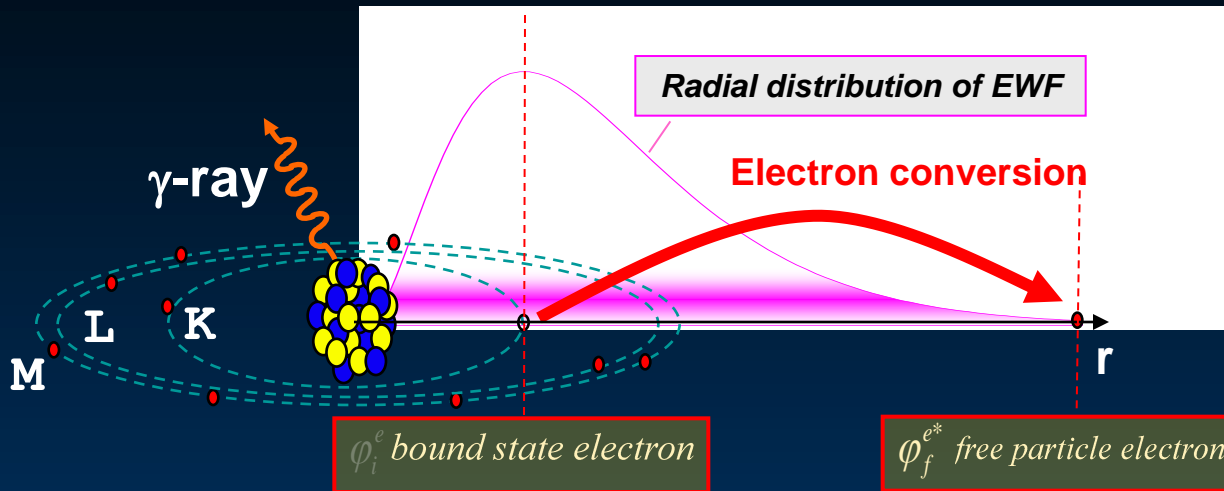
Density of the final electron state (continuum)



# ICC calculations – Atomic field model

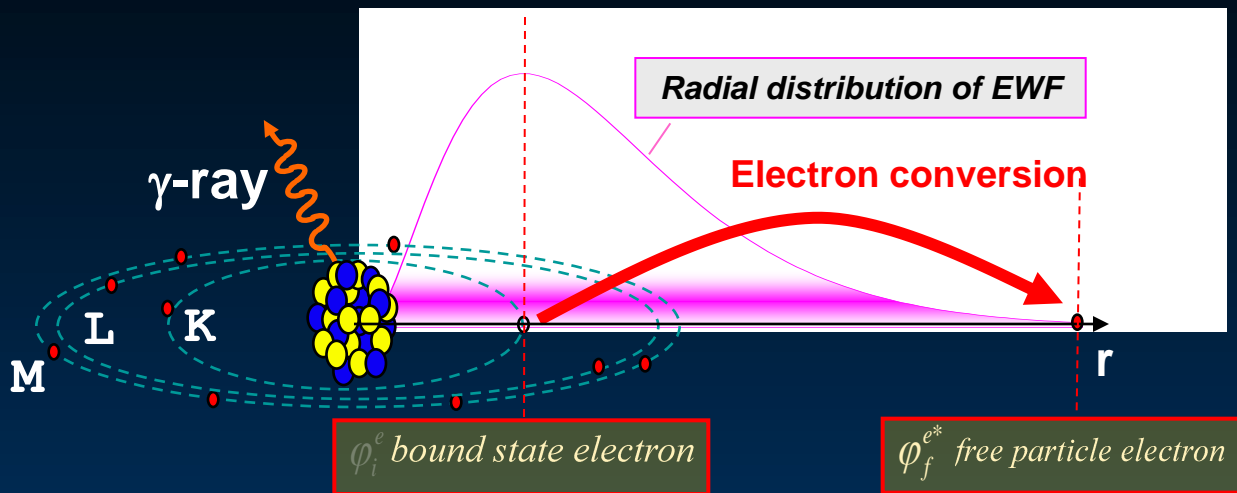


- Relativistic Dirac-Fock method
- One-electron approximation
- Free neutral atom
- Screening of the nuclear field by the atomic electrons
- Spherically symmetric atomic potential
- Relativistic electron wave functions
- Experimental electron binding energies



- Finite nuclear size
- Dynamic (penetration) effects incorporated using the Surface Current model
- Spherically symmetric nucleus; most abundant isotope

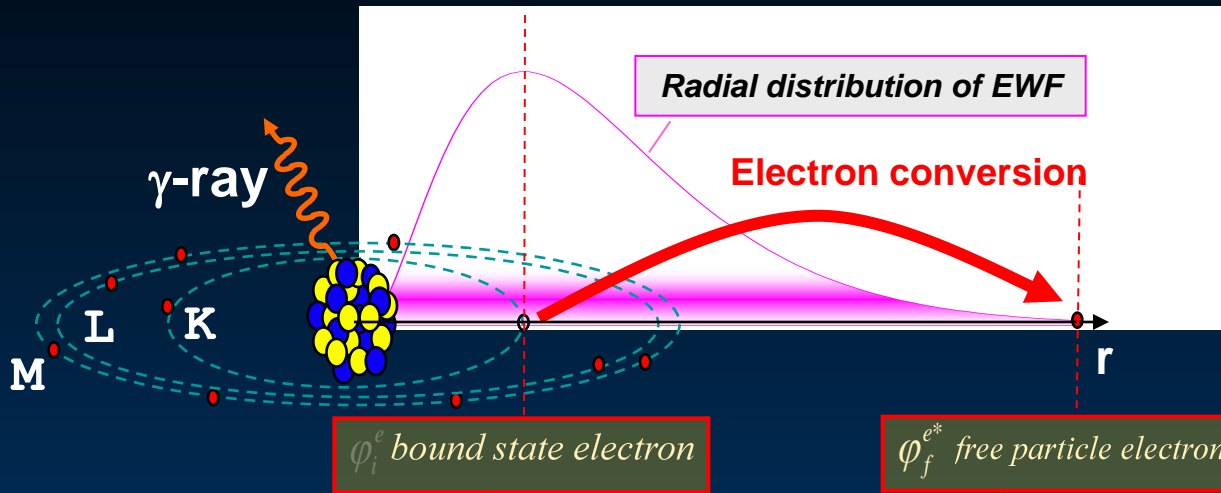
# ICC calculations – Higher order effects



- Atomic many body correlations: factor  $\sim 2$  for  $E_{\text{kin}}(\text{ce}) < 1 \text{ keV}$
- Partially filled valence shell: non-spherical atomic field
- Binding energy uncertainty:  $< 0.5\%$  for  $E_{\text{kin}}(\text{ce}) > 10 \text{ keV}$
- Chemical effects:  $\ll 1\%$



# ICC calculations – Atomic vacancies



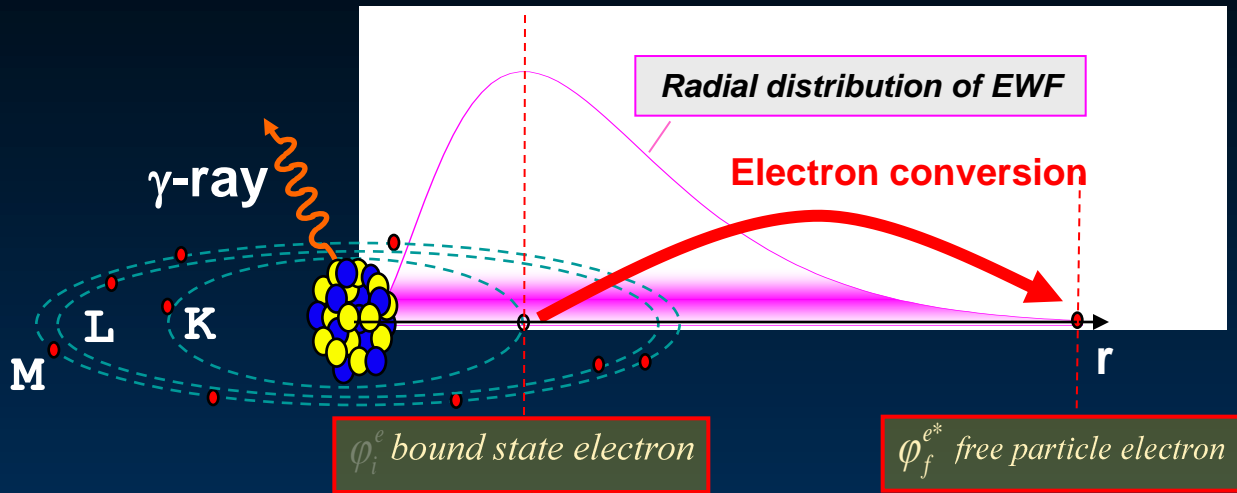
"No Hole" - BTNTR: SCF of a neutral atom

Vacancy disregarded

2002Ba85 numerical tables  
(Band et al., ADNDT 81 (2002) 1)

BrIccNH table - extended and revised calculations

# ICC calculations – Atomic vacancies

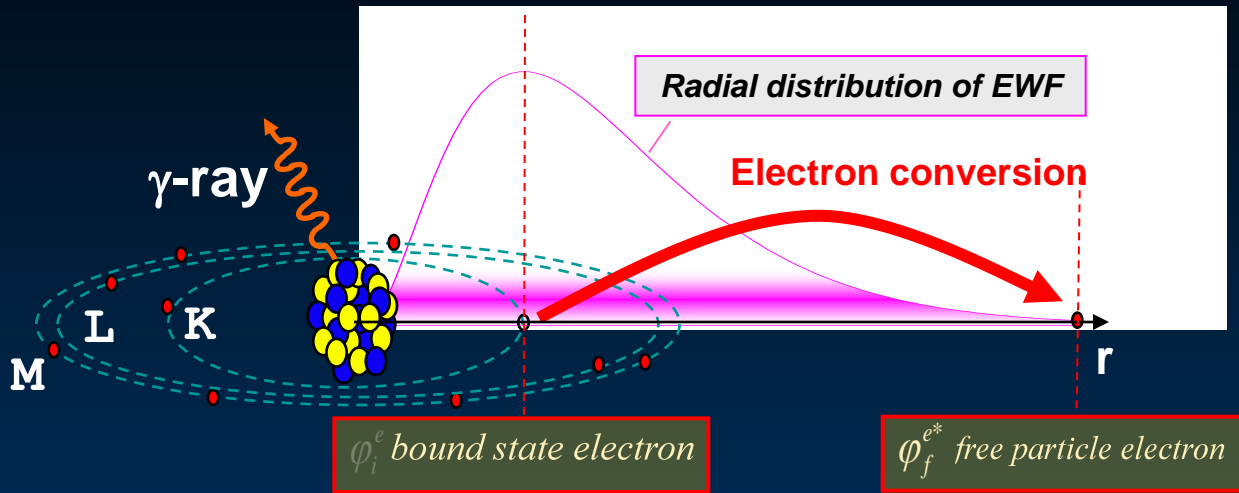


SCF of a neutral atom      SCF of an ion  
*"Self Consistent" - RNIT(1)*

*Vacancy incorporated*

*No data table*

# ICC calculations – Atomic vacancies



SCF of a neutral atom

Constructed from the  
WF of a neutral atom,  
not SCF

"Frozen Orbitals" – RNIT(2)

Vacancy incorporated  
*BrIccFO data table*

*Kibedi et al., Nucl. Instr. and Meth. A 589 (2008) 202*

# Which atomic model to choose

ML	Shell	N	"No Hole" BTNTR		<i>"Self Consistent"</i> <i>RNIT(1)</i>		<i>"Frozen Orbital"</i> RNIT(2)	
				$\chi^2/(N-1)$		$\chi^2/(N-1)$		$\chi^2/(N-1)$
All	All	229	+ 0.70(40)	1.82	-0.61(14)	1.01	-0.93(14)	0.87
Tot	All	63	+ 0.32(25)	0.79	-0.55(24)	0.76	-0.71(24)	0.73
K	All	91	+ 1.50(120)	3.14	-0.18(21)	1.09	-0.72(21)	0.80
K/L	All	56	+ 0.00(31)	0.83	-1.64(31)	0.96	-1.94(30)	1.02
E2	All	115	+ 0.21(23)	1.01	-0.77(23)	0.89	-0.93(23)	0.90
M4	All	49	+ 0.98(68)	3.87	-0.51(20)	1.29	-0.93(20)	0.72
ICCs known better than 1.5% rel. unc.								
All	All	25	+ 0.77(51)	8.21	-0.56(26)	2.12	-0.95(17)	1.06

- **General tool to obtain**
  - **electron conversion coefficients:  $\alpha_i$ ;  $i=K, L1, L2, \dots R2$  shells;  $L=1-5$**
  - **electron-positron pair conversion coefficients:  $L=1-3$**
  - **E0 electronic factors:  $\Omega_i (K,L1,L2)$  and  $\Omega_\pi$**
- **ENSDF evaluation tool to calculate:**
  - ▶  **$\alpha_i \pm \Delta\alpha_i$  for the GAMMA records for a given  $Z, E_\gamma$  and for pure or mixed multipolarities.**
  - ▶ **Correct treatment of Uncertainties in E and MR; symmetric, asymmetric or limits**
  - ▶ **GAMMA & GAMMA continuation records scanned for E, DE, M, MR, DMR, RI, DRI, TI & DTI, CC, DCC**
  - ▶ **Calculation report**
  - ▶ **New G cards with CC and DCC values**
  - ▶ **New S\_G cards for other applications**
  - ▶ **MERGE option to combine new cards with ENSDF file, old records removed**

# BrlccFO(Default) and BrlccNH data tables

Data Table	Reference	Z	Shells/IPF	L	TranEner [keV] <sup>a</sup>
<i>Internal Conversion Coefficient (ICC)</i>					
<b>BrlccFO</b>	<a href="#">2008Ki07</a> - based on the model using the 'Frozen Orbitals' approximation of <a href="#">2002Ba85</a>	5–110	All shells	1–5	$\epsilon_{ic}+1-6000$
<b>BrlccNH</b>	<a href="#">2008Ki07</a> - based on the model using the 'No Hole' approximation of <a href="#">2002Ba85</a>	5–110	All shells	1–5	$\epsilon_{ic}+1-6000$
<i>Pair Conversion Coefficient (PCC)</i>					
<b>ScPcc</b>	<a href="#">1979Sc31</a>	0–100 <sup>b</sup>	IPF	1–3	1100–8000
<b>HoPcc</b>	<a href="#">1996Ho21</a>	50–100	IPF	1–3	1100–8000
<i>Electronic factor <math>\Omega(E0)^c</math></i>					
<b>HsOmg</b>	<a href="#">1969Ha61</a>	30–42	$K^d, L_1^e, L_2^f$	0	$\epsilon_{ic}+6-1500$
<b>BeOmg</b>	<a href="#">1970Be87</a>	40–102	K	0	51 <sup>f</sup> –2555
		40–102	$L_1, L_2$	0	51–2555
<b>PaOmg</b>	<a href="#">1986PaZM</a>	8–40	$K^e$	0	511–12775
		8–40	IPF	0	1431–12775

<sup>a</sup>  $\epsilon_{ic}$  is the binding energy for the ic-shell

<sup>b</sup> Used for  $Z < 50$

<sup>c</sup> Electronic factors are only calculated for even Z values at present

<sup>d</sup> Not used

<sup>e</sup> Used for  $Z < 40$

<sup>f</sup> For  $Z=40-58$ : 51.1 keV; for  $Z=60-82$ : 102.2 keV; for  $Z=84-96$ : 153.3 keV and for  $Z=98-102$ : 204.4 keV

# BrIcc – Recent developments

Version	Date	Description
0.0	15-Nov-2003	Project initiated at the <a href="#">NSDD coordination meeting</a> (Vienna)
1.0	6-Apr-2004	Tabulations using the ' <i>No Hole</i> ' approximation completed for $\epsilon_{ic}+1-6000$ keV energies and Z=10-95 atomic numbers.
1.3	20-Dec-2004	BrIcc released for ENSDF evaluators.
	6-Jun-2005	<a href="#">NSDD coordination meeting</a> (McMaster) adopted the ' <i>Frozen Orbitals</i> ' approximation.
2.0	1-Sep-2005	Tabulations using the ' <i>Frozen Orbitals</i> ' approximation completed. BrIcc program logic, exception and error handling improved.
2.1	1-Oct-2005	BrIccS (' <i>slave</i> ') version developed and the new ANU web interface created.
2.0a	19-Dec-2005	Corrected a bug in the BrIcc <i>MERGE</i> operation.
2.0b	12-Jan-2006	Corrected a bug in the BrIcc <i>MERGE</i> operation.
2.1	28-Feb-2007	User selectable data tables, including <i>BrIccFO</i> , <i>BrIccNH</i> , <i>HsIcc</i> and <i>RpIcc</i> were added to the ANU web interface.
2.2	4-Jan-2008	The <i>BrIccFO</i> and the <i>BrIccNH</i> data tables have been recalculated for several chemical elements and the range of the tables have been extended for Z=5-110.
2.2a	13-Jul-2008	The lowest tabulated point in the <i>BrIccFO</i> and the <i>BrIccNH</i> data tables for Z=88, 98, 100, 101 and 102 K-shell was 2 keV above the binding energies.
	27-Nov-2008	Minor bug in the Slave version fixed. For pure and mixed ICC`s and for transition energy within 1 keV to the binding energy incorrect XML code was returned.
2.2b	20-Jan-2009	For Z=34 incorrect $\Omega_{IPF}(E0)$ values were used for the PaOmg data table.

# Brlcc (v2.3) – Current programs

- Intel Fortran compiler on Windows, Linux, Mac
- Improvements in the code, fixed several bugs
- 32/64 bit
- BrIcc - ENSDF evaluation tool / desktop application to obtain ICC` s
- BrIccS - slave program to be used from other programs (DDEP, Web)
- BrIccG - to chart ICC values vs. transition energy
- BrIccMixing - to deduce mixing ratio from CE data
- BrIccEmis - under development



# BrIcc (v2.3) evaluation tool – new features

If no MR is given use NEW DEFAULT values of

MR=1 for E2/M1 and E3/M2

MR=0.1 for M2/E1, M3/E2, E4/M3, M5/E4, M4/E3  
and E5/M4

$MREOE2 = EK(E0)/EK(E2)$  experimental E0/E2 mixing ratio

BrIcc will read from Continuation G records

If  $CC > 0.0001$  total ICC will be put on G record (56-64)


Otherwise CC will be put on the S\_G record

NEW: Let user select cut-off value of CC

<http://bricc.anu.edu.au/index.php>

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## BrIcc v2.3G

Conversion Coefficient Grapher

Z (atomic number or symbol)

**Multipolarity**  
  $\delta$

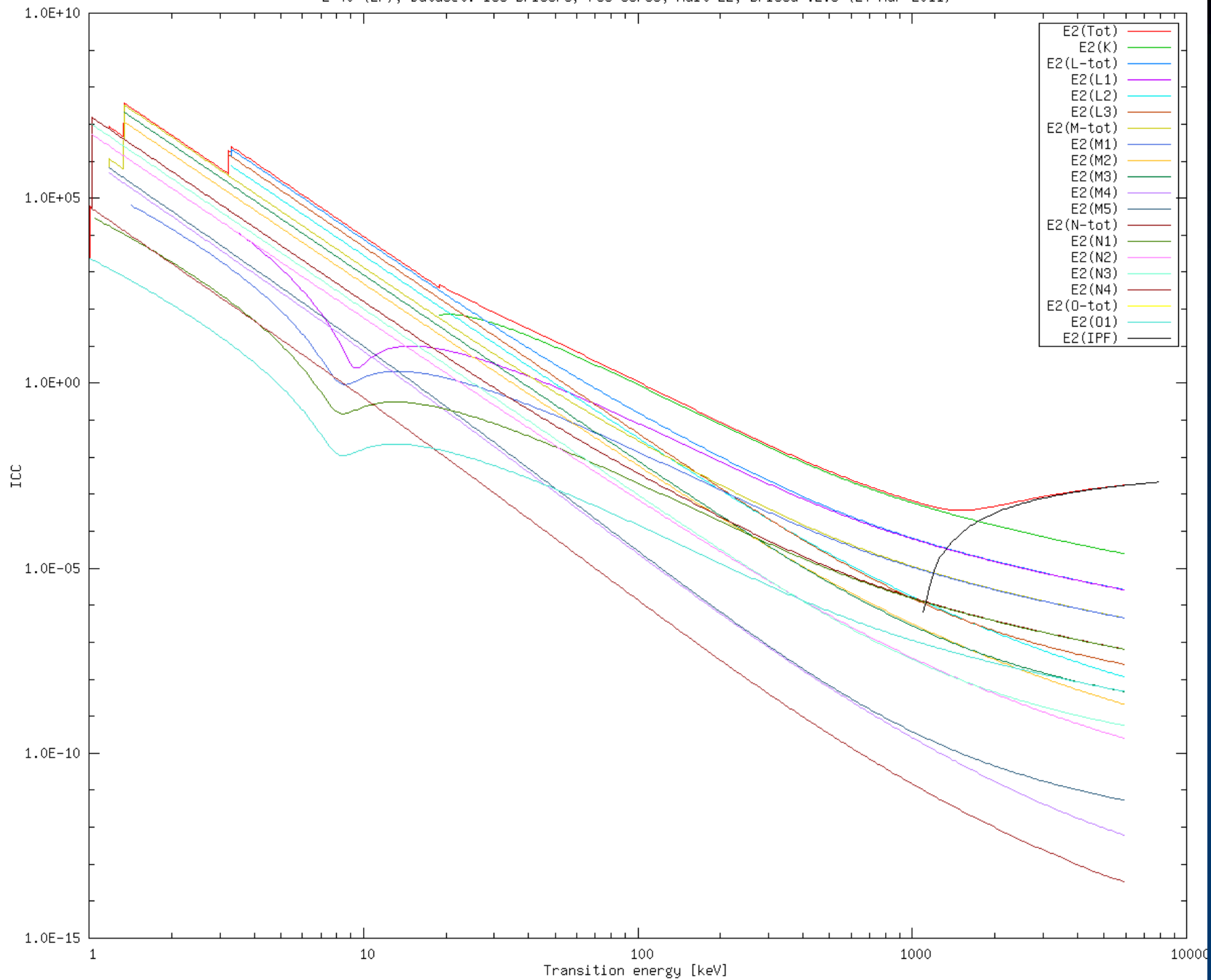
Enter (optional) uncertainty in  $\delta$  as **x** or **+x-y**

**Shell or Ratio**  
 Shell  **OR** Shell1  **over** Shell2

**Show Subshells**


(It may take up to 30 seconds for the graph to be created)

Reference:  
[2008KI07](#) T. Kibédi, T.W. Burrows, M.B. Trzhaskovskaya, P.M. Davidson, C.W. Nestor, Jr.  
*'Evaluation of theoretical conversion coefficients using BrIcc'*  
 Nucl. Instr. and Meth. A 589 (2008) 202-229



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## BrIcc v2.3G

Conversion Coefficient Grapher

Z (atomic number or symbol)

**Multipolarity**  
     $\delta$

Enter (optional) uncertainty in  $\delta$  as **x** or **+x-y**

**Shell or Ratio**  
Shell  OR Shell1  over Shell2

Show Subshells

(It may take up to 30 seconds for the graph to be created)

Reference:  
[2008KI07](#) T. Kibédi, T.W. Burrows, M.B. Trzhaskovskaya, P.M. Davidson, C.W. Nestor, Jr.  
*'Evaluation of theoretical conversion coefficients using BrIcc'*  
Nucl. Instr. and Meth. A 589 (2008) 202-229

**With** M.B. Trzhaskovskaya, M. Gupta and A.E. Stuchbery  
**Accepted in ADNDT**

## **Existing ICC tables**

- Rysavy and Dragoun, ADNDT 78 (2001) 129  
Dirac-Slater atomic model, few energy points
- Band et al., ADNDT 81 (2002) 1  
Relativistic Dirac-Fock atomic model, “No-Hole”  
approximation, from L1 shell binding energy

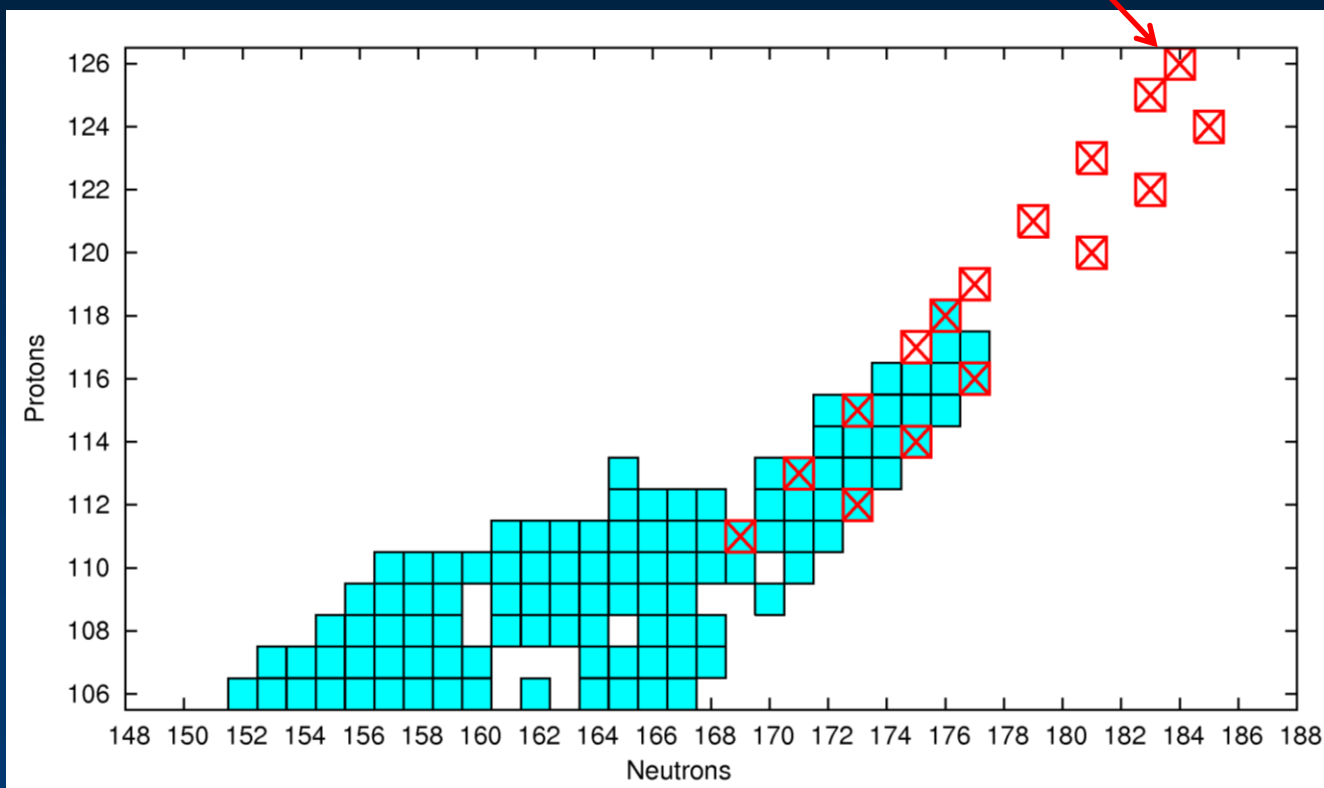
### Input parameters

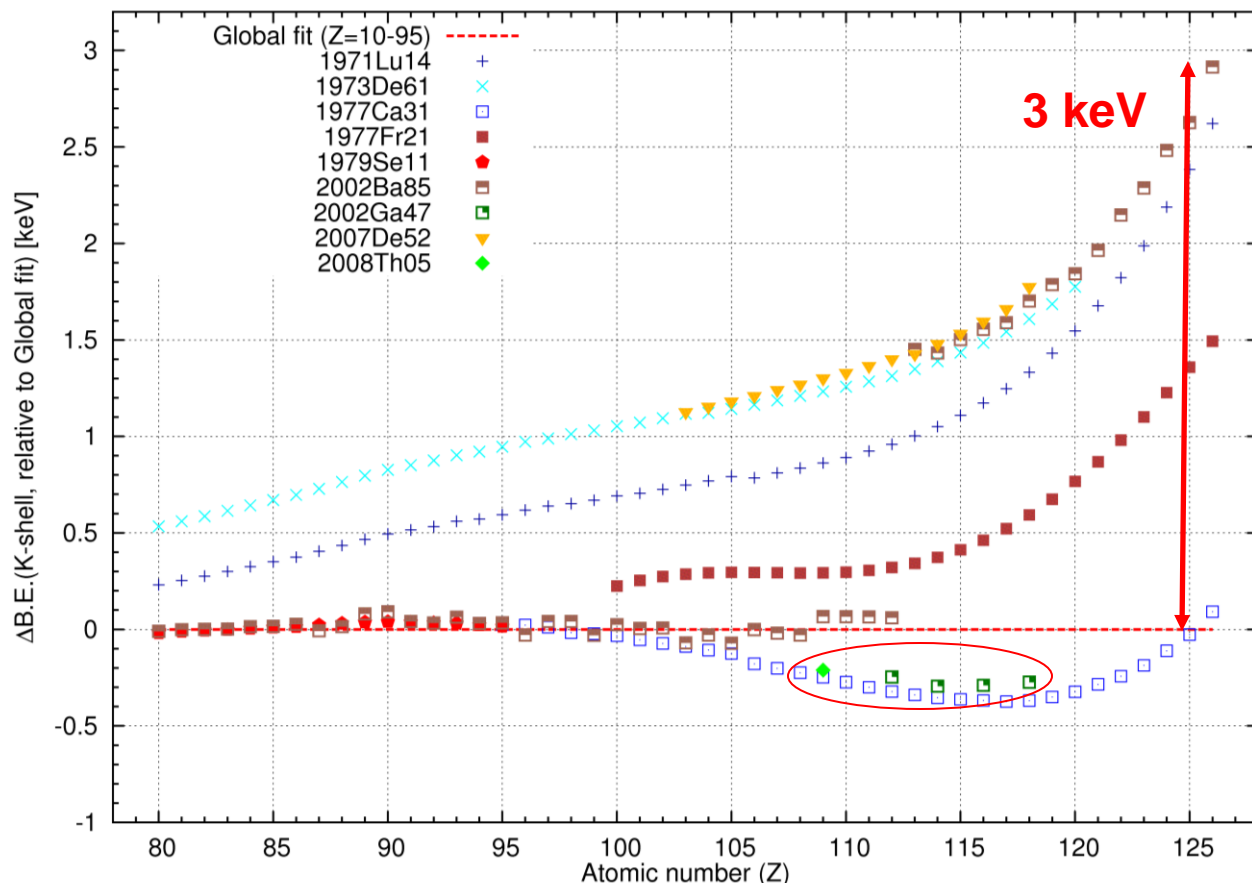
- Adopted atomic masses
- Valence electron configurations
- Neutral atom binding energies

Next shell closure at  
Z=114, 120 or 126  
N=184

IUPAC Atomic Weights  
and Isotopic Abundances:  
Z=111-116, 118

Z=117 new isotopes  
2010Og01





## Global fit

1979Se11 Sevier Z=10-95  
2002Ba85 Band et al. Z>95

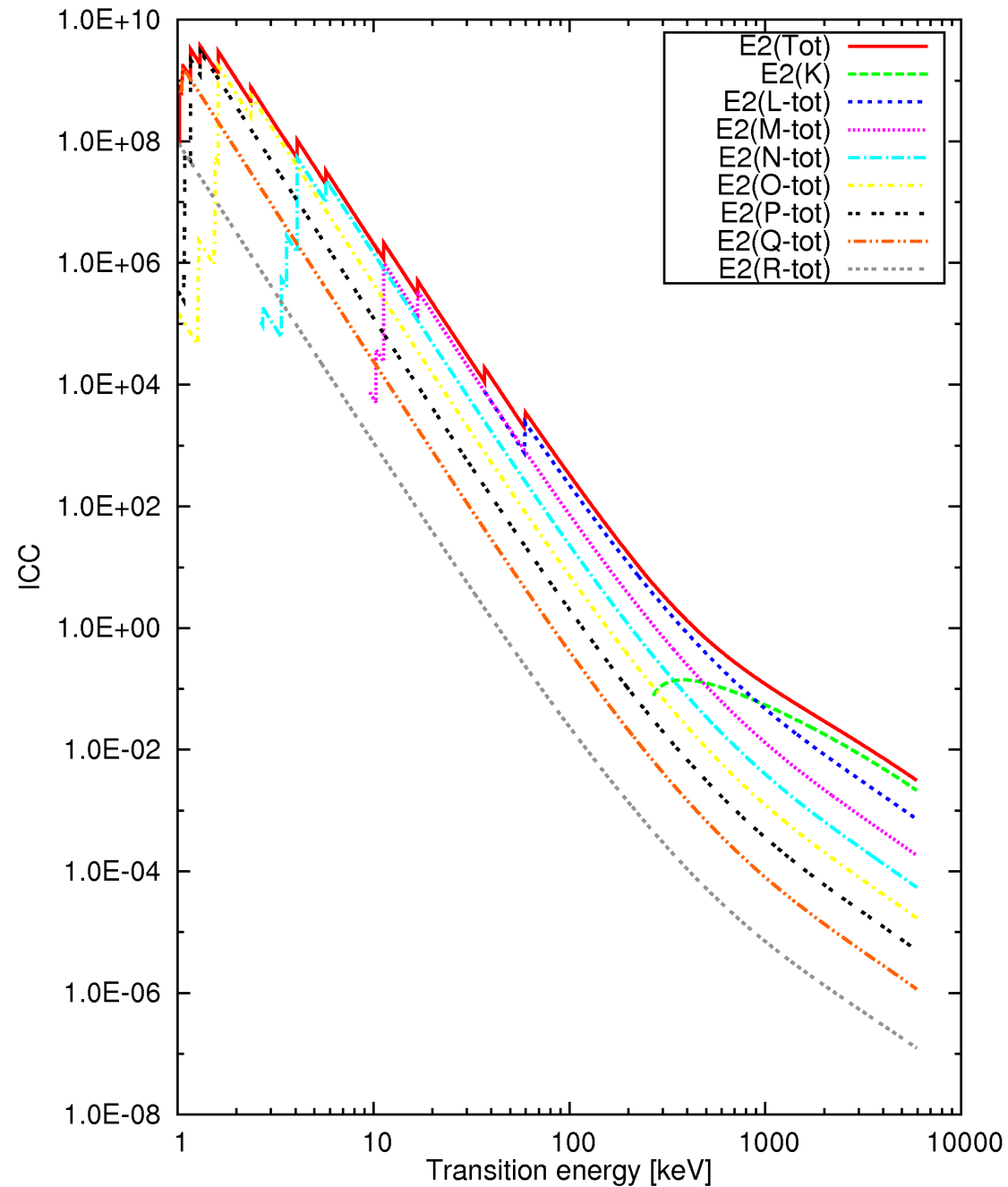
Adopted  
1977Ca31 Carlson & Nestor  
relativistic Dirac-Fock with  
semi-empirical corrections

New systematic  
calculations needed!

New relativistic Dirac-Fock calculations with QED corrections  
(2002Ga47, 2007De52)

**Z=126**  
**E2 multipolarity**  
**1 – 6000 keV**  
**All shells**  
**E1-E5 M1-M5**

**More electron shells used**  
**Not compatible with current**  
**BrIcc**





Shell	$E_{tr}$ [keV]	ICC(M1)				ICC(E2)			
		NICC [24]	Brlcc V1.3 [8]	BrlccNH <sup>(a)</sup>	BrlccFO <sup>(b)</sup>	NICC [24]	Brlcc V1.3 [8]	BrlccNH <sup>(a)</sup>	BrlccFO <sup>(b)</sup>
Total	10	1.883E+4	–	1.670E+4	1.673E+4	2.248E+6	–	2.070E+6	2.072E+6
	50	5.128E+2	–	4.647E+2	4.650E+2	4.555E+3	–	4.368E+3	4.374E+3
	100	2.445E+2	2.24E+2	2.238E+2	2.243E+2	3.419E+2	3.30E+2	3.300E+2	3.319E+2
	300	3.075E+1	2.84E+1	2.841E+1	2.852E+1	3.552E+0	3.48E+0	3.475E+0	3.493E+0
	1000	9.807E-1	8.82E-1	8.814E-1	8.817E-1	1.236E-1	1.19E-1	1.193E-1	1.199E-1
K	270	–	–	2.554E+1	2.576E+1	–	–	7.434E-2	7.895E-2
	300	2.079E+1	1.92E+1	1.919E+1	1.929E+1	1.079E-1	1.04E-1	1.107E-1	1.154E-1
	1000	6.639E-1	5.99E-1	5.984E-1	5.986E-1	5.526E-2	5.37E-2	5.392E-2	5.441E-2
L1	61	–	5.13E+2	5.153E+2	5.183E+2	–	5.27E+1	5.337E+1	5.513E+1
	100	1.323E+2	1.21E+2	1.214E+2	1.217E+2	1.115E+1	1.09E+1	1.091E+1	1.110E+1
	300	5.262E+0	4.89E+0	4.890E+0	4.895E+0	3.532E-1	3.50E-1	3.500E-1	3.525E-1
	1000	1.648E-1	1.48E-1	1.479E-1	1.479E-1	1.970E-2	1.92E-2	1.915E-2	1.920E-2
M1	18	–	–	4.896E+3	4.919E+3	–	–	4.155E+3	4.240E+3
	50	2.597E+2	–	2.365E+2	2.367E+2	7.018E+1	–	6.772E+1	6.834E+1
	100	3.328E+1	3.04E+1	3.037E+1	3.040E+1	4.956E+0	4.82E+0	4.817E+0	4.845E+0
	300	1.320E+0	1.22E+0	1.215E+0	1.215E+0	1.170E-1	1.15E-1	1.151E-1	1.154E-1
	1000	4.110E-2	3.66E-2	3.658E-2	3.658E-2	5.300E-3	5.12E-3	5.117E-3	5.121E-3
N1	7	–	–	2.473E+4	2.483E+4	–	–	1.298E+5	1.311E+5
	10	9.595E+3	–	8.555E+3	8.572E+3	2.704E+4	–	2.518E+4	2.541E+4
	50	7.841E+1	–	7.063E+1	7.063E+1	2.569E+1	–	2.457E+1	2.466E+1
	100	1.004E+1	9.09E+0	9.063E+0	9.066E+0	1.702E+0	1.64E+0	1.639E+0	1.643E+0
	300	3.984E-1	3.62E-1	3.620E-1	3.620E-1	3.783E-2	3.67E-2	3.673E-2	3.678E-2
	1000	1.237E-2	1.09E-2	1.088E-2	1.089E-2	1.627E-3	1.55E-3	1.554E-3	1.554E-3

## NICC

Rysavy and Dragoun,  
ADNDT 78 (2001) 129  
Systematically larger by  
5-10%

**BrlccV1.3** (2002Ba85)  
Band et al., ADNDT 81  
(2002) 1

Up to 5% for E2 in K  
and L1

*BrlccFO* / *BrlccNH*  
“Frozen orbitals” vs.  
“No-Hole”

Up to 5% for E2 in K  
and L1

## E0 Transition probability

$$W_{ic}(E0) = \rho^2(E0) \times \Omega_{ic}(E0)$$

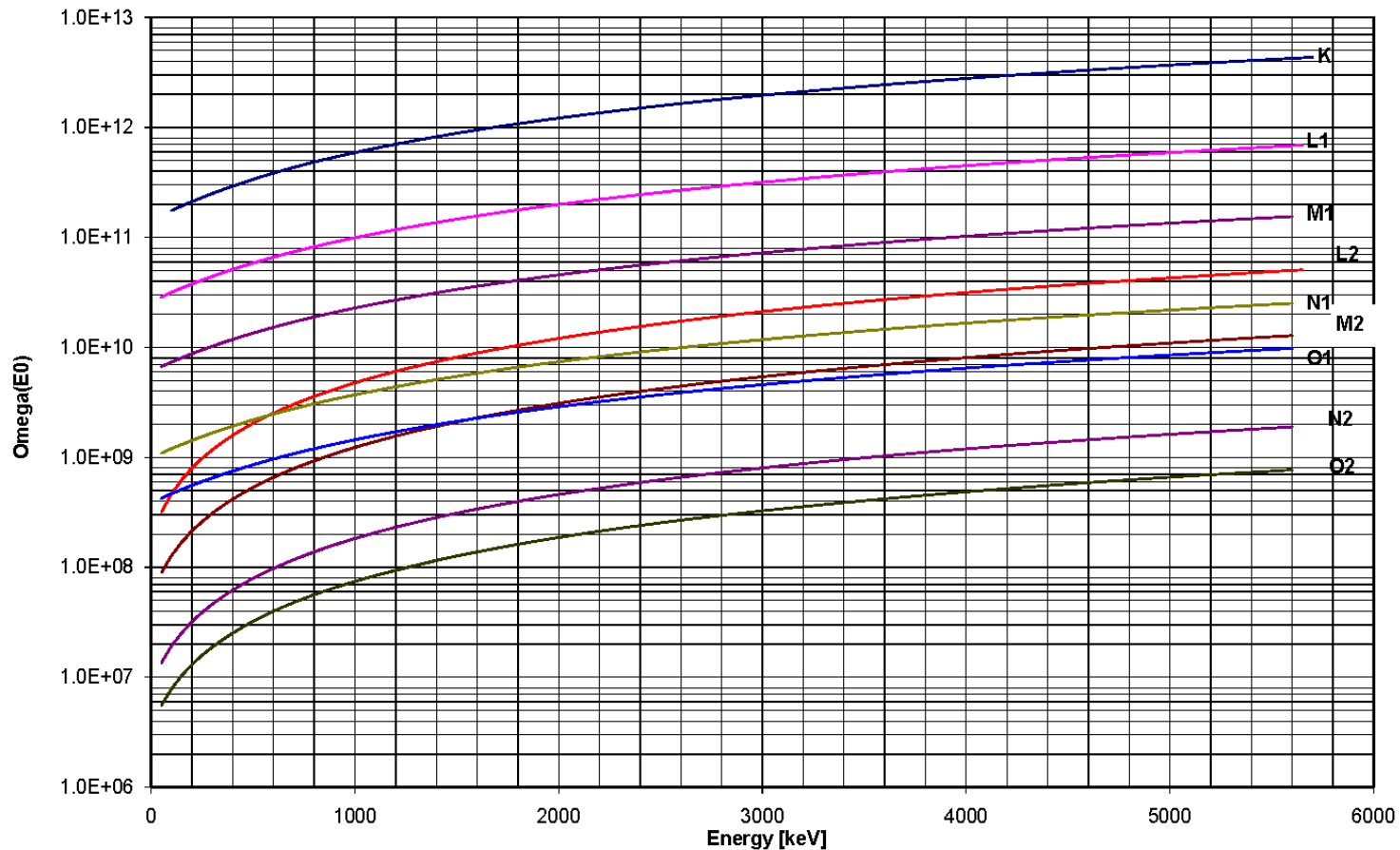
Nuclear                      Atomic

### Current BrIcc Table

- composed from 3 tabulations
- Do not cover all energies and atomic shells
- $\Omega(E0)$  and ICC tables are incompatible

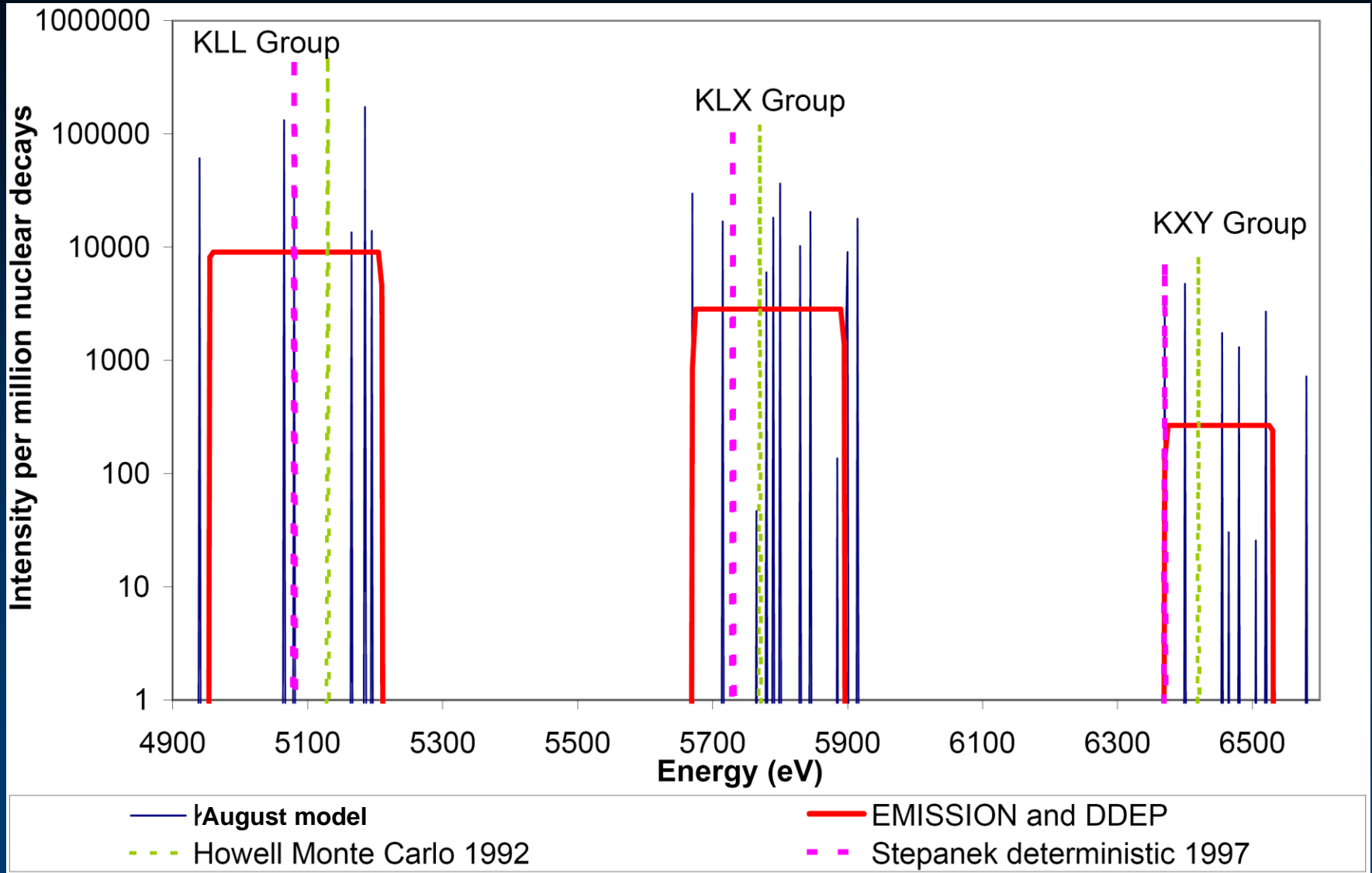
### New calculations using CATAR (H.C.Pauli, U.Raff 1975Pa26)

# Z=82 E0 electronic factors



Need to compare with existing tabulations and ratios of sub-shell CE intensities of E0 transitions (K/L, L1/L2)

# $^{55}\text{Fe} + e^- \rightarrow ^{55}\text{Mn} + \nu$ K-Auger electrons (5 eV bins)

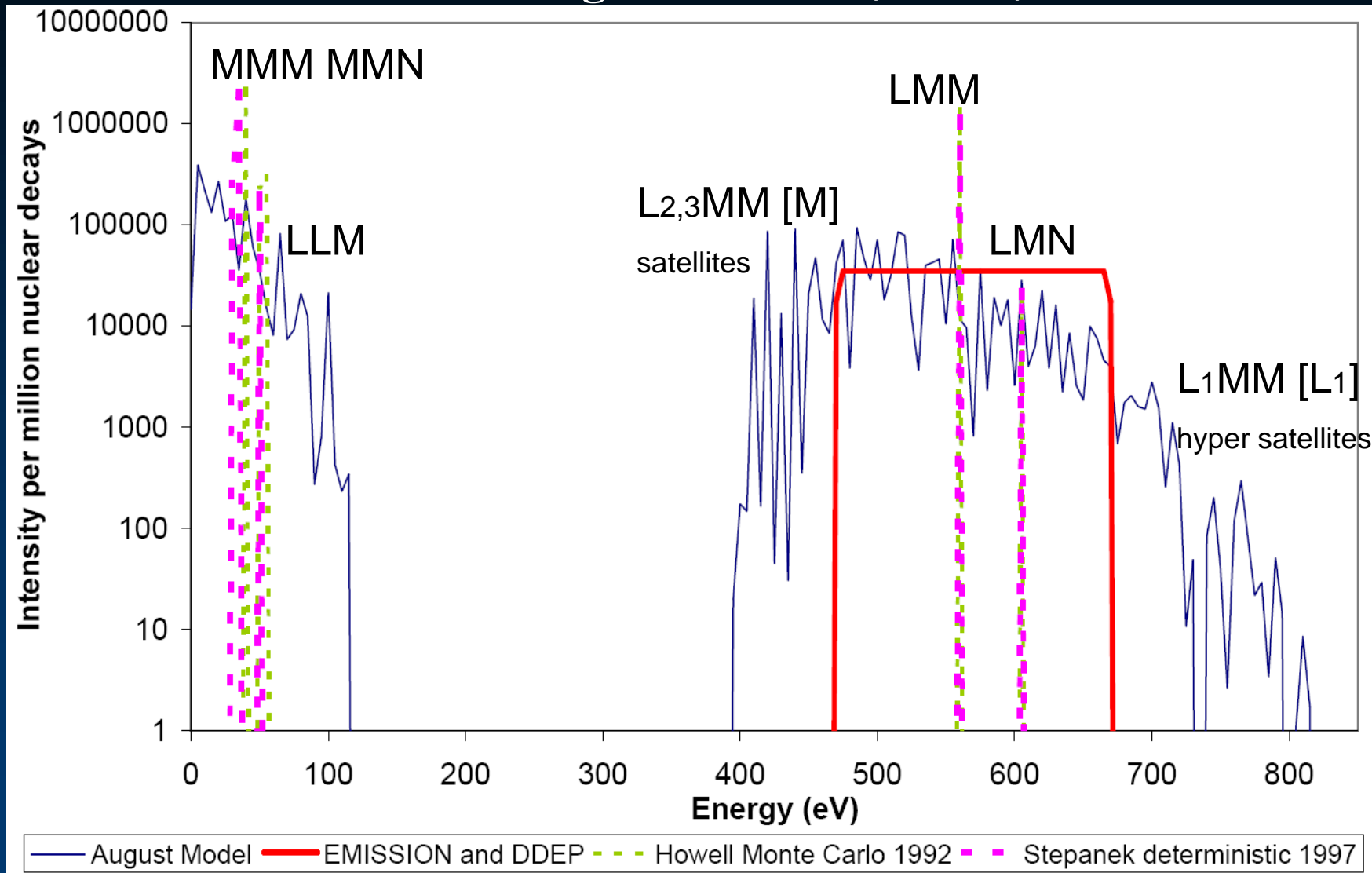




$3.7 \times 10^6$  Auger electrons and  $2.8 \times 10^5$  X-rays per  $10^6$  nuclear decays



### L- and M- Auger electrons (5 eV bins)



**Table 6.1:** Mean energy and intensity (per million nuclear disintegrations) of groups of atomic radiations in nuclear decay of  $^{55}\text{Fe}$  in condensed phase. The August Model results are calculated using Approach 2 as explained in Section 6.3. For each transition group in the August Model results, the uncertainty in the mean energies is less than 30 eV and the uncertainty in intensity is less than 5%. The DDEP values agree with the EMISSION values except for the *LXY* transition intensity (indicated by \*), which is  $1.40 \text{ E } 6$  according to DDEP and  $1.31 \text{ E } 6$  by running the EMISSION program. “Bound-bound X-rays” and “free-bound X-rays” are abbreviated “b-b X” and “f-b X.” The bound-bound X-ray, Auger, CK and free-bound X-ray totals are the total energies released in these respective forms per million nuclear decays. The average energy released through atomic transitions per nuclear decay is  $(5890 \pm 50) \text{ eV}$  (see section 6.6).

**Kalman Robertson, Honours Thesis, ANU, October 2010**

Radiation	August Model		EMISSION/DDEP		Howell		Stepanek	
	Energy	Intensity	Energy	Intensity	Energy	Intensity	Energy	Intensity
$K_{\alpha}$	5925 eV	2.492E5	5895 eV	2.50E5	5895 eV	2.44E5	5860 eV	2.44E5
$K_{\beta}$	6530 eV	3.389E4	6512 eV	3.40E4	6490 eV	2.97E4	6450 eV	2.91E4
<i>L</i> b-b X	665 eV	4.340E3	565 eV	4.90E3	635 eV	3.30E3	585 eV	6.90E3
<i>M</i> b-b X	50 eV	< 1E2	N/A	N/A	N/A	N/A	N/A	N/A
b-b X total	1701 MeV		1700 MeV		1636 MeV		1566 MeV	
$KLL$	5100 eV	4.212E5	5080 eV	4.66E5	5130 eV	4.87E5	5080 eV	4.87E5
$KLX$	5790 eV	1.653E5	5780 eV	1.27E5	5770 eV	1.20E5	5730 eV	1.19E5
$KXY$	6440 eV	1.469E4	6450 eV	8.62E3	6420 eV	8.20E3	6370 eV	7.30E3
$LXY$	513 eV	1.346E6	570 eV	1.40E6*	564 eV	1.43E 6	561 eV	1.42E6
$LLX$	44 eV	3.563E5	N/A	N/A	55.2 eV	3.10E5	31.4 eV	2.63E5
<i>M</i> and <i>N</i> Auger/CK	23 eV	1.396E6	N/A	N/A	42 eV	2.77E6	38 eV	2.83E6
Auger/CK total	3938 MeV		3950 MeV		4170 MeV		4240 MeV	
f-b X	40 eV	4.699E6	N/A	N/A	N/A	N/A	11 eV	6.13E6
f-b X total	189 MeV		N/A		N/A		68.3 MeV	